Evidence of the silver ratio[1] in financial market time series data

"The time element as an independent device, however, continues to be baffling when attempts are made to apply any known rule of sequence to trend duration."
-- RN Elliott, Nature's Law[2]

As the first in an infinite set of silver means[3], the golden ratio[4] occupies a heralded position. Does this justify the primogeniture of mindshare it enjoys today? Are its silver mean siblings so much less remarkable?

Disclaimers

- Many or most people view many or most aspects of technical analysis as kindred to imagining shapes of farm animals in cloud formations. These pages are not intended to disavow anyone of such a notion. Rather, for the consumption of those who know the joy of alchemizing dollars from patterns or are curious about extra-fundamental analyses, they are offered as a stepping stone (however meager in the attendant footing) to a more thorough investigation of how markets function.
- Elliott Wave analysis[5] is a contentious topic. Anyone who finds this approach distasteful should note that despite assorted references, acceptance of the theory is in no way a pre-requisite to consideration of the irrational (in the strict mathematical sense) relationships evidenced in the included charts. So, if waves aren't your flavor, purely perusing **Pictures** is probably a prudent maneuver.
- Academically and professionally, my knowledge and experience in matters discussed here are woefully incomplete. If anyone is aware of un-cited work with comparable, complementary, or contradictory findings to any that follow, I would very much appreciate being alerted to it. Gratitude--research@rationalinsolvency.com

Keywords

dynamic symmetry, Elliott, Fibonacci, Gann, golden ratio, Pell, silver means, silver ratio, socionomics, technical analysis

Blather

In espousing the Wave Principle[6], Elliott didn't only revolutionize market analysis, but also how human interaction can be conceptualized[7]. While Elliott explicitly claimed he was not aware of the Fibonacci sequence during his early formulations[8], by the time his final writings on the subject were completed, the golden ratio came to play a celebrated, almost deified, role in the narrative[9,i,10].

Following his death, decades of relatively little popular attention to Elliott's ideas were ended with a resurgence of the theory in the late 1970s[11,ii]. As popularity increased during the years that ensued, more

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[i] In Chapter II of Nature's law, Elliott cites a lengthy passage from Hambidge[10] which focuses upon φ. Hambidge's earlier 1920 work, Dynamic Symmetry: the Greek Vase takes care to acknowledge the role of $\sqrt{2}$ and, in turn, δ_s, in the dynamic symmetries observed in the works of antiquity.

[ii] Even the Fibonacci Quarterly, after publishing a sparse summary of Elliott's work in their second year (1964, with the part II continuation the following year), chose not to include any financial market related studies thereafter (among the publicly available back-issues). The still active newsletter has evolved toward greater exclusivity regarding mathematical theory, but in years past,

concrete and practically-oriented interpretations and extensions of Elliott's original work emerged[12,13]. Similar efforts have continued to the present day[14]. While there is no question these derivatives satisfy a demand, they have also served to obscure some of Elliott's more philosophical contentions.

Recently, there has been a rabbit-like multiplication of interest in the application of Fibonacci arithmetic to financial markets. The first 13 result pages of a Google Books search for "Fibonacci analysis" yields no fewer than 89 titles specific to financial markets published during the last 8 years. By contrast, while not offering a single title related to financial markets, a search for "Pell analysis" returns (presumptively) invaluable resources to assist in securing federally subsidized US student financial aid packages. Searching for "golden ratio" leads unambiguously to phi. The most pertinent results for "silver ratio" regard Au divided by Ag. Thanks to Tom Hanks and Ron Howard, there is no shortage of Americans who, despite having long forgotten the bulk of their High School Geometry, are familiar with the golden ratio[15]. The point of these observations is not to disparage the first silver mean, but merely to illustrate that it has developed a healthy set of publicists. Amidst this phi-xation, I wonder if maybe we've lost perspective.

Foundations

The Fibonacci sequence can be defined as:

$F(n) = \{0 \text{ for } n=0,\ 1 \text{ for } n=1;\ F(n-1) + F(n-2) \text{ for } n>1\}$

$= 0,1,1,2,3,5,8,13,21,...$

The quotient of consecutive Fibonacci numbers approximates the golden ratio, φ:

$\varphi = \frac{1}{2}(1 + \sqrt{5}) \approx 1.6180$

$1/\varphi \approx 0.6180$

The Pell sequence can be defined as:

$P(n) = \{0 \text{ for } n=0;\ 1 \text{ for } n=1;\ 2P(n-1) + P(n-2) \text{ for } n>1\}$

$= 0,1,2,5,12,29,70,169,408,...$[iii,16]

The quotient of consecutive Pell numbers approximates the silver ratio, δ_s:

$\delta_s = 1 + \sqrt{2} \approx 2.4142$

$1/\delta_s \approx 0.4142$

Pictures

During the two plus months from early May to early July, with an error of about 1 hour, the low between the $SPX 1370 and 1356 peaks is isolated using silver ratio-based proportions of the entire duration.

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it has had repeated contributions on topics in the fields of biology, chemistry, computer science, electrical engineering, game theory, and optics. Selected articles on casino betting have also appeared. Of particular note to the topic at hand, the Fibonacci Quarterly has published a variety of pieces on or related to Pell numbers.

[iii] Note that for $n<8$, $\text{FLOOR}[\sqrt{P(n)}] = F(n)$. Perhaps this relationship provides basis for Gann's notion of "squaring" price and time and testifies to the appearance of Fibonacci time relationships in markets. $\sqrt{P(8)} \approx 20.1990 < F(8) = 21$ = the integer average number of trading days in a month. Gann's 1x1 and 1x2 angles befit the silver and golden ratios with their shared relation to $\sqrt{(1^2+1^2)}$ and $\sqrt{(1^2+2^2)}$.

Figure 1[17]

Silver ratio related subdivision is also apparent within those two segments of price action (aka at the next lower Elliott degree; Elliott labels have been omitted both to illustrate methodological independence and to spare virtuous Elliotticians from the nausea induced by my counts).

Figure 2

Further subdivision is possible, but coarse bar granularity serves to obscure the elegance of the underlying proportions.

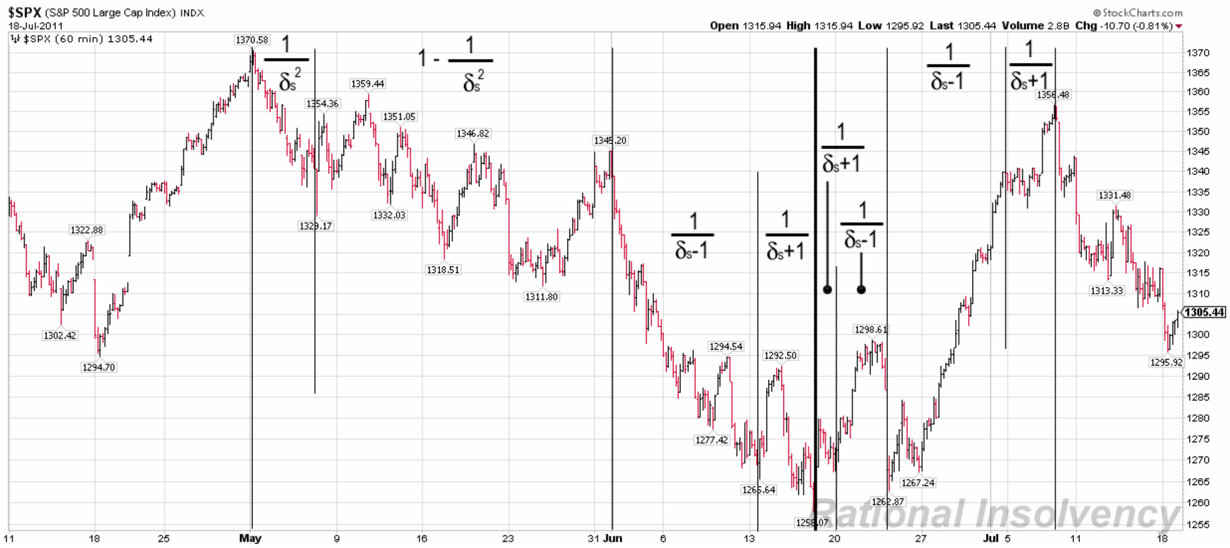

Figure 3

The fractal nature of markets implies that further subdivision is always possible--the only hurdle is sampling at a sufficiently high frequency. That is, if you believe in that sort of thing.

It should be noted that Fibonacci analysis is not without its applicability to the case at hand.

Figure 4

Counting daily bars, the decline spanned 32 and the ensuing uptrend 14. Those numbers are very near F(9) and F(7), so a maximum of two days error would have been achieved if looking for Fibonacci durations. More skillful Fibonacci practitioners undoubtedly have more impressive indications, but what might that suggest about silver ratio time analysis, given its potential for accuracy in such lowly skilled hands?

Relationships between alternating up and down moves also exhibit proportionality consistent with the silver ratio.

Figure 5

The italicized integers in Figure 5 refer to the number of trading days during which each up or down move occurred. This silver ratio proportionality between the moves of differing direction has the largest divergence from the ideal of any of the Figures presented--an error of 1.2997%. Note that using the nearest popular golden ratio based proportion, $1 - 1/\varphi^3$, yields 6.2197% error. Alternatively, F(3)/F(4) is off by 7.4627%. The ratio of days containing up waves (48) relative to the sum of all measures (48+67=115) differs from the ideal value of $1/\delta_s$ by 0.0032, or 0.7672%.

The last segment of price action in Figure 5 also reveals the silver ratio internally.

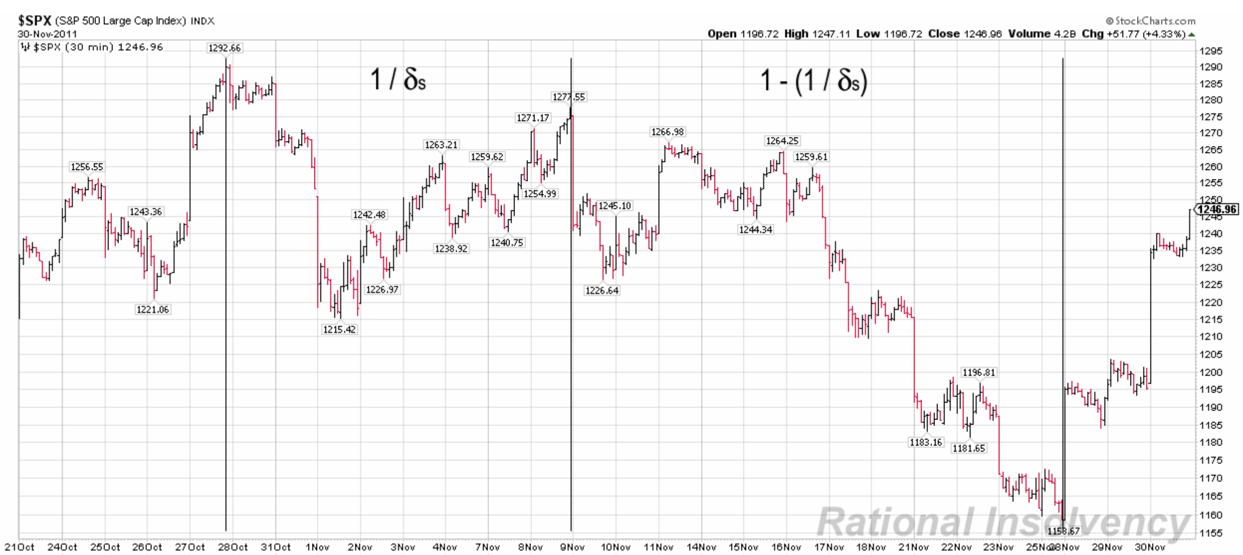

Figure 6

Notice how nicely the 1277.55 peak separates the larger move from 1292.66 to 1158.67 into proportions consistent with the second silver mean.

Had the similar Fibonacci related 0.382 been used instead of 0.414, the pattern would instead have been expected to have ended 10 hours later. The closing price of that bar was 3.5% higher than the 1158.67 trough.

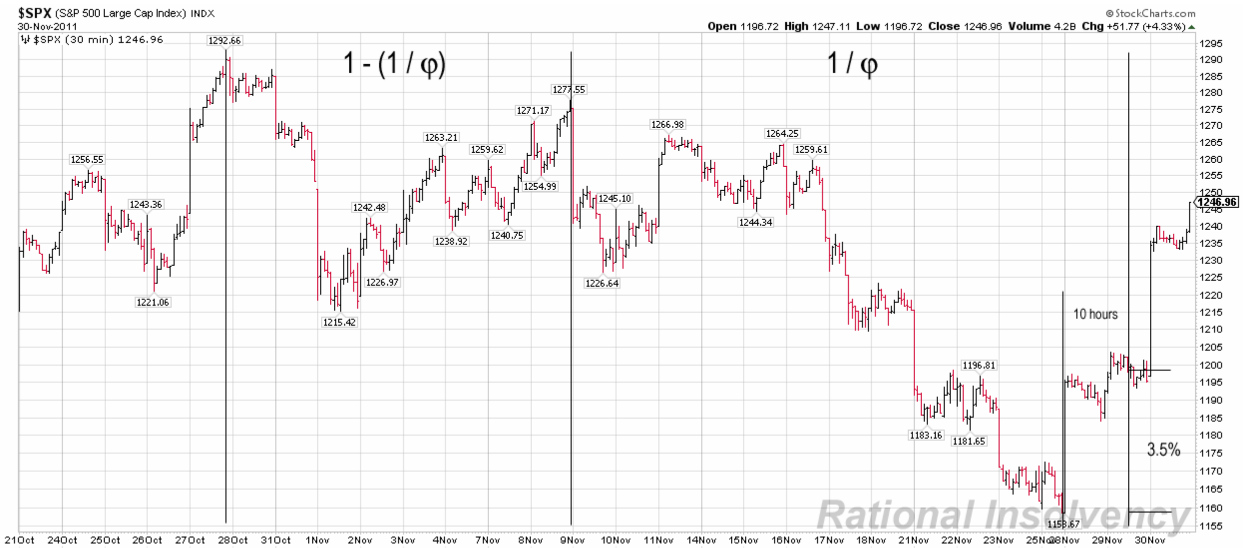

Figure 7

In retrospect, the analyst in this particular case may have salved that 3.5% with some Fibonacci numbers.

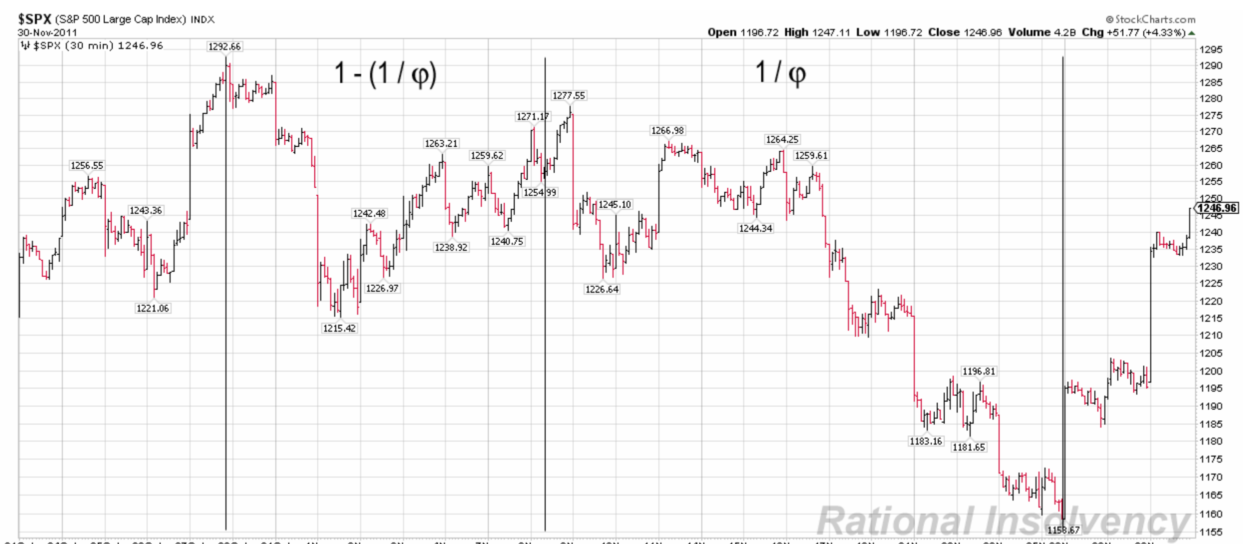

Figure 8

Figure F(6) illustrates how adjusting the total duration to reality and realigning the 0.382 marker yields a position F(6) bars prior to the 1277.55 high. The whole of the move from 1292.66 to 1158.67 lasted approximately F(8) days.

There is no sufficient number of examples that could be presented to "prove" this idea. For those who believe market data to be truly fractal, the above may even be overly tedious. Regardless, if additional evidence is desired, it isn't hard to find. All sorts of examples are available daily and the process is identical to Fibonacci analysis, only with a different irrational constant. Nonetheless, this may be met with displeasure by some who are accustomed to phi--I'm not trying to pick a fight; just offering some observations.

Kookiness

Of course, the identification of silver ratio proportions within and between the duration of Elliott waves may be nothing more than trivial coincidence. This sort of argument may sound familiar to early adopters of Fibonacci analysis. The Fibonacci sequence's applicability to not just numbers of Elliott waves, but also a variety of natural forms is well documented[18]. Is there any reason to believe the Pell sequence has any pertinence to measures of time? (Admittedly, I've witnessed much less than the following paragraph dismissed as numerological bias confirmation, so continue at your own peril of reason.)

There are P(3)=5 trading days in a week[iv]. There are P(4)=12 months in a year. The regular session of the typical New York Stock exchange trading day lasts six and a half hours, or P(7) / 26[v]. Is 26 an integer as arbitrary as the rest in this paragraph? It isn't quite the number of days in the mean sidereal month[19], mean hormonal cycle[20], or mean synodic month[21]; although, P(5)=29 is a reasonable integer approximation of the latter two. Measured in weeks, 26 is a half year. As 5 is to 7, 26 is to 36.4--very nearly 1/10th the average number of days in a calendar year. There are other oddities in the power of 10, as the number of minutes in a trading week, 1950 is almost exactly 1/100th of P(15)=195025 (as well as roughly twice P(9)=985). These associations may only be bounded by our imaginations and I know there are plenty better than mine who might keep ringing the Pell.

As a sociometer, financial market data comes with the baggage of a two-dimensional representation in time and price. Where is it written in stone that the dimensionality of the artifacts of maiesthai[22] is necessarily so limited?

Elliott noted that market volume data adheres to the pattern of the Wave Principle. Could the proportionality of its fluctuations be consistent with some third silver mean?

Is there additional market data which might move in a manner consistent with yet other silver means?

Should the following have been added as a fourth sentence at the outset: Are its metallic mean[23] cousins also less remarkable?

Technicians have long used volume data to assist in identification of price/time patterns. Some silver means are relatable to each other via basic algebra (see Appendix B). Does this imply there is a more formal basis for relating various dimensions of manifestations social mood?

If the individual dimensions of social mood show traces consistent with successive silver means, might the circle of quiet contemplation in Kapusta's Figure 46[24] have a relation to (or even model) the progress of social mood? What relationships can we divine if the basic shape is not a circle, but a sphere?

Before considering any possible answers to the preceding, skepticism confronts me about whether the *questions* are even useful. Hopefully other people will come up with much better ones, but, until then, I'll take lousy queries over false certainty 8 days a week.

[iv] I'm punting on any resultant implication regarding social mood during non-market days.

[v] Note, apropos footnote *iii*, P(7) = 169 = 13^2 = $F(7)^2$. Fittingly, in turn, the Fibonacci-Pell arithmetic breadth has declined in *v* relative to *iii*. Alternation is also apparent between *ii* and *iv*. Will *v* extend? The ratio of chars in *iii* to *v* is φ.

When you unravel an onion and it makes you cry, it might prove that you're dealing with an onion. Or it may just confirm that you're too emotionally attached to unraveling. If anyone has noticed mistakes in the forgoing, be they grand or subminuette, I'd appreciate a head's up. Thanks-research@rationalinsolvency.com

[1] http://en.wikipedia.org/wiki/Silver_ratio
[2] R.N. Elliott's Masterworks: The Definitive Collection, Prechter (ed), 192
[3] http://en.wikipedia.org/wiki/Silver_mean
[4] http://en.wikipedia.org/wiki/Golden_ratio
[5] http://en.wikipedia.org/wiki/Elliott_wave
[6] R.N. Elliott's Masterworks: The Definitive Collection, Prechter (ed), 85
[7] R.N. Elliott's Masterworks: The Definitive Collection, Prechter (ed), 225
[8] R.N. Elliott's Masterworks: The Definitive Collection, Prechter (ed), 192
[9] R.N. Elliott's Masterworks: The Definitive Collection, Prechter (ed), 218
[10] Practical Applications of Dynamic Symmetry, Hambidge
[11] Elliott Wave Principle, Frost & Prechter
[12] Elliott Wave Principle Applied to the Foreign Exchange Markets, Balan
[13] Mastering Elliott Wave, Hall & Neely
[14] http://www.harmonic-ewave.com
[15] http://www.imdb.com/title/tt0382625/
[16] http://en.wikipedia.org/wiki/W._D._Gann
[17] Un-annotated charts provided courtesy www.stockcharts.com
[18] The Wave Principle of Human Social Behavior and the New Science of Socionomics, Prechter, Chapter 3
[19] http://en.wikipedia.org/wiki/Sidereal_month#Sidereal_month
[20] http://en.wikipedia.org/wiki/Menstrual_cycle#Length
[21] http://en.wikipedia.org/wiki/Synodic_month#Synodic_month
[22] The Wave Principle of Human Social Behavior and the New Science of Socionomics, Prechter, 18
[23] http://www.mi.sanu.ac.rs/vismath/spinadel/index.html
[24] http://www.scipress.org/journals/forma/pdf/1904/19040293.pdf

Appendix A: Potentially useful numbers

$S_1(n) = F(n) = \{0 \text{ for } n=0, 1 \text{ for } n=1; F(n-1) + F(n-2) \text{ for } n>1\}$
$= 0,1,1,2,3,5,8,13,21,...$

$\varphi = \frac{1}{2}(1+\sqrt{5})$	≈ 1.6180
$\sqrt{\varphi}$	≈ 1.2720
φ^2	≈ 2.6180
φ^3	≈ 4.2361
$1/\varphi = \varphi-1$	≈ 0.6180
$1/\sqrt{\varphi}$	≈ 0.7862
$1/\varphi^2 = 1-(1/\varphi)$	≈ 0.3820
$1/\varphi^3$	≈ 0.2361
$1-(1/\varphi^3)$	≈ 0.7639
$F(2)/F(3)$	$= 0.5000$

$S_2(n) = P(n) = \{0 \text{ for } n=0; 1 \text{ for } n=1; 2P(n-1) + P(n-2) \text{ for } n>1\}$
$= 0,1,2,5,12,29,70,169,408,...$

$\delta_s = 1 + \sqrt{2}$	≈ 2.4142
$\sqrt{\delta_s}$	≈ 1.5538
δ_s^2	≈ 5.8284
δ_s^3	≈ 14.0711
$1/\delta_s = \delta_s-2$	≈ 0.4142
$1/\delta_s^2$	≈ 0.1716
$1/\sqrt{\delta_s}$	≈ 0.6436
$P(1)/P(2)$	$= 0.5000$
$1-(1/\delta_s)$	≈ 0.5858
$1/(\delta_s-1) = \frac{1}{2}(\delta_s-1)$	≈ 0.7071
$1/(\delta_s+1) = \frac{1}{2}(1-(1/\delta_s))$	≈ 0.2929

$S_3(n) = \{0 \text{ for } n=0; 1 \text{ for } n=1; 3S_3(n-1) + S_3(n-2) \text{ for } n>1\}$
$= 0,1,3,10,33,109,360,1189,3927,...$

$S_3 = \frac{1}{2}(3 + \sqrt{13})$	≈ 3.3028
$\sqrt{S_3}$	≈ 1.8174
S_3^2	≈ 10.9083
S_3^3	≈ 36.0278
$1/S_3$	≈ 0.3028
$1/S_3^2$	≈ 0.0917
$1/\sqrt{S_3}$	≈ 0.5503
$S_3(2)/S_3(3)$	$= 0.5000$
$1-(1/S_3)$	≈ 0.6972

Appendix B: Table of the first P(5) silver means

$S_n = \frac{1}{2}[n+\sqrt{(n^2+4)}]$		$S_{2n} = n+\sqrt{(n^2+1)}$		
$S_1 = \frac{1}{2}(1+\sqrt{5})$				5 is prime
$S_2 = \frac{1}{2}(2+\sqrt{8})$	$= \frac{1}{2}(2+2\sqrt{2})$	$= 1+\sqrt{2}$		
$S_3 = \frac{1}{2}(3+\sqrt{13})$				13 is prime
$S_4 = \frac{1}{2}(4+\sqrt{20})$	$= \frac{1}{2}(4+2\sqrt{5})$	$= 2+\sqrt{5}$	$= 2S_1+1$	
$S_5 = \frac{1}{2}(5+\sqrt{29})$				29 is prime
$S_6 = \frac{1}{2}(6+\sqrt{40})$	$= \frac{1}{2}[6+2\sqrt{(2*5)}]$	$= 3+\sqrt{10}$		10 = F(3) * F(5)
$S_7 = \frac{1}{2}(7+\sqrt{53})$				53 is prime
$S_8 = \frac{1}{2}(8+\sqrt{68})$	$= \frac{1}{2}(8+2\sqrt{17})$	$= 4+\sqrt{17}$		17 = F(7) + F(4) + F(2) 17 = P(4) + P(3)
$S_9 = \frac{1}{2}(9+\sqrt{85})$				85 = F(5) * 17
$S_{10} = \frac{1}{2}(10+\sqrt{104})$	$= \frac{1}{2}[10+2\sqrt{(2*13)}]$	$= 5+\sqrt{26}$		26 = F(3) * F(7)
$S_{11} = \frac{1}{2}(11+\sqrt{125})$	$= \frac{1}{2}(11+5\sqrt{5})$		$= \frac{1}{2}(5S_4+1) = 5S_1+1$	$125 = F(5)^3$
$S_{12} = \frac{1}{2}(12+\sqrt{148})$	$= \frac{1}{2}(12+2\sqrt{37})$	$= 6+\sqrt{37}$		37 = F(9) + F(4) 37 = P(5) + P(3) + P(2) + P(1)
$S_{13} = \frac{1}{2}(13+\sqrt{173})$				173 is prime
$S_{14} = \frac{1}{2}(14+\sqrt{200})$	$= \frac{1}{2}(14+10\sqrt{2})$	$= 7+5\sqrt{2}$	$= 5S_2+2$	
$S_{15} = \frac{1}{2}(15+\sqrt{229})$				229 is prime
$S_{16} = \frac{1}{2}(16+\sqrt{260})$	$= \frac{1}{2}[16+2\sqrt{(5*13)}]$	$= 8+\sqrt{65}$		65 = F(5) * F(7)
$S_{17} = \frac{1}{2}(17+\sqrt{293})$				293 is prime
$S_{18} = \frac{1}{2}(18+\sqrt{328})$	$= \frac{1}{2}[18+2\sqrt{(2*41)}]$	$= 9+\sqrt{82}$		82 = F(10) + F(8) + F(5) + F(2) 82 = P(6) + P(4)
$S_{19} = \frac{1}{2}(19+\sqrt{365})$				365 = F(5) * 73
$S_{20} = \frac{1}{2}(20+\sqrt{404})$	$= \frac{1}{2}(20+2\sqrt{101})$	$= 10+\sqrt{101}$		101 is prime
$S_{21} = \frac{1}{2}(21+\sqrt{445})$				445 = F(5) * F(11)
$S_{22} = \frac{1}{2}(22+\sqrt{488})$	$= \frac{1}{2}[22+2\sqrt{(2*61)}]$	$= 11+\sqrt{122}$		122=F(11)+F(8)+F(6)+F(4)+F(2) 122 is not a sum of P(n)'s
$S_{23} = \frac{1}{2}(23+\sqrt{533})$				533 = F(7) * 41
$S_{24} = \frac{1}{2}(24+\sqrt{580})$	$= \frac{1}{2}[24+2\sqrt{(5*29)}]$	$= 12+\sqrt{145}$		29 = P(5)
$S_{25} = \frac{1}{2}(25+\sqrt{629})$				629 = F(7) * 17
$S_{26} = \frac{1}{2}(26+\sqrt{680})$	$= \frac{1}{2}[26+2\sqrt{(2*34)}]$	$= 13+\sqrt{170}$		170 = 2 * 5 * 17 = F(5) * F(9)
$S_{27} = \frac{1}{2}(27+\sqrt{733})$				733 is prime
$S_{28} = \frac{1}{2}(28+\sqrt{788})$	$= \frac{1}{2}(28+2\sqrt{197})$	$= 14+\sqrt{197}$		197 is prime
$S_{29} = \frac{1}{2}(29+\sqrt{845})$	$= \frac{1}{2}(29+13\sqrt{5})$		$= 13S_1+8 = \frac{1}{2}(13S_4+3)$ $= 1/5 * (13S_{11}+1)$	$845 = 5 * 13^2 = F(5) * F(7)^2$ $= P(3) * P(7)$

Within the first P(5) silver means, select silver means are linearly expressible using rational coefficients. More specifically, within the first $P(F(P(F(P(2)^2))))$ silver means, those expressible by other silver means can be represented as:
$S_n = [F(x)]^{-1} * [F(y)*S_m + F(z)]$ for integers x,y,z
That Lenny is one crafty fella.

Appendix C: Fun with four digits.

Take the following silver ratio based pairs, rounded to four places:
0.4142, 0.5858
0.7071, 0.2929

A tautological statement:
0.4142 + 0.5858 = 1 = 0.7071 + 0.2929

Notice,
0.5858 = 2 * 0.2929
0.7071 = 0.4142 + 0.2929

Each of 0.4142 and 0.7071 can be represented as 0.(n)(n+1) with n=41 and 70, respectively.
Each of 0.5858 and 0.2929 can be represented as 0.(n)(n) with n=58 and 29, respectively.

P(6)=70, P(5)=29

Take one other silver ratio based pair, rounded as the prior two:
0.1716, 0.8284

0.1716 can be represented as 0.(n)(n-1) with n=17
0.8284 can be represented as 0.(n)(n+2) with n=82

Incidentally,
(n-1) + (n+2) = 2n + 1 = (n+1) + (n)

And,
0.8284 = 2 * 0.4142

70 - 41 = 29 = P(5)
58 - 41 = 17
70 - 58 = 41 - 29 = 82 - 70 = 29 - 17 = 12 = P(4)

Notice the numbers in the far right column in Appendix B. Primes are uncharacteristically multiple. Fibonacci numbers recur. P(5)=29 rings twice. 17 = F(9)/2 and the first two digits of $1/\delta_s^2$. 82 is a companion Pell, Q(5) as well as double the first two digits of $1/\delta_s$; 41 also makes an appearance in the factorization of the irrational part of S_{23}. 122 is double the first two digits of $1/\varphi$. But what of 37?